

Main Ideas

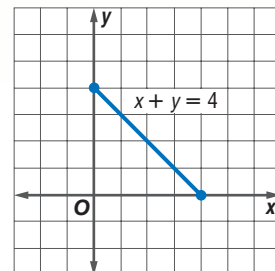
- Identify linear equations and functions.
- Write linear equations in standard form and graph them.

New Vocabulary

linear equation
linear function
standard form
 y -intercept
 x -intercept

GET READY for the Lesson

Lolita has 4 hours after dinner to study and do homework. She has brought home math and chemistry. If she spends x hours on math and y hours on chemistry, a portion of the graph of the equation $x + y = 4$ can be used to relate how much time she spends on each.



Identify Linear Equations and Functions An equation such as $x + y = 4$ is called a linear equation. A **linear equation** has no operations other than addition, subtraction, and multiplication of a variable by a constant. The variables may not be multiplied together or appear in a denominator. A linear equation does not contain variables with exponents other than 1. The graph of a linear equation is always a line.

Linear equations

$$5x - 3y = 7$$

$$x = 9$$

$$6s = -3t - 15$$

$$y = \frac{1}{2}x$$

Not linear equations

$$7a + 4b^2 = -8$$

$$y = \sqrt{x + 5}$$

$$x + xy = 1$$

$$y = \frac{1}{x}$$

A **linear function** is a function whose ordered pairs satisfy a linear equation. Any linear function can be written in the form $f(x) = mx + b$, where m and b are real numbers.

EXAMPLE Identify Linear Functions

1 State whether each function is a linear function. Explain.

- $f(x) = 10 - 5x$ This is a linear function because it can be written as $f(x) = -5x + 10$. $m = -5$, $b = 10$
- $g(x) = x^4 - 5$ This is not a linear function because x has an exponent other than 1.
- $h(x, y) = 2xy$ This is not a linear function because the two variables are multiplied together.

CHECK Your Progress

1A. $f(x) = \frac{5}{x + 6}$

1B. $g(x) = -\frac{3}{2}x + \frac{1}{3}$



Real-World Link

To avoid decompression sickness, it is recommended that divers ascend no faster than 30 feet per minute.

Source: www.emedicine.com



Real-World EXAMPLE

Evaluate a Linear Function

2

WATER PRESSURE The linear function $P(d) = 62.5d + 2117$ can be used to find the pressure (lb/ft²) d feet below the surface of the water.

- a. Find the pressure at a depth of 350 feet.

$$P(d) = 62.5d + 2117 \quad \text{Original function}$$

$$P(350) = 62.5(350) + 2117 \quad \text{Substitute.}$$

$$= 23,992 \quad \text{Simplify.}$$

The pressure at a depth of 350 feet is about 24,000 lb/ft².

- b. The term 2117 in the function represents the atmospheric pressure at the surface of the water. How many times as great is the pressure at a depth of 350 feet as the pressure at the surface?

Divide the pressure 350 feet down by the pressure at the surface.

$$\frac{23,992}{2117} \approx 11.33 \quad \text{Use a calculator.}$$

The pressure at that depth is more than 11 times that at the surface.



CHECK Your Progress

2. At what depth is the pressure 33,367 lb/ft²?



Personal Tutor at algebra2.com

Standard Form Many linear equations can be written in **standard form**, $Ax + By = C$, where A , B , and C are integers whose greatest common factor is 1.

KEY CONCEPT

Standard Form of a Linear Equation

The standard form of a linear equation is $Ax + By = C$, where A , B , and C are integers whose greatest common factor is 1, $A \geq 0$, and A and B are not both zero.

EXAMPLE

Standard Form

3

Write each equation in standard form. Identify A , B , and C .

- a. $y = -2x + 3$

$$y = -2x + 3 \quad \text{Original equation}$$

$$2x + y = 3 \quad \text{Add } 2x \text{ to each side.}$$

So, $A = 2$, $B = 1$, and $C = 3$.

- b. $-\frac{3}{5}x = 3y - 2$

$$-\frac{3}{5}x = 3y - 2 \quad \text{Original equation}$$

$$-\frac{3}{5}x - 3y = -2 \quad \text{Subtract } 3y \text{ from each side.}$$

$$3x + 15y = 10 \quad \text{Multiply each side by } -5 \text{ so that the coefficients are integers and } A \geq 0.$$

So, $A = 3$, $B = 15$, and $C = 10$.



CHECK Your Progress

3A. $2y = 4x + 5$

3B. $3x - 6y - 9 = 0$



Extra Examples at algebra2.com

Study Tip

Vertical and Horizontal Lines

An equation of the form $x = C$ represents a vertical line, which has only an x -intercept. $y = C$ represents a horizontal line, which has only a y -intercept.

Since two points determine a line, one way to graph a linear equation or function is to find the points at which the graph intersects each axis and connect them with a line. The y -coordinate of the point at which a graph crosses the y -axis is called the **y -intercept**. Likewise, the x -coordinate of the point at which it crosses the x -axis is the **x -intercept**.

EXAMPLE Use Intercepts to Graph a Line

- 4 Find the x -intercept and the y -intercept of the graph of $3x - 4y + 12 = 0$. Then graph the equation.

The x -intercept is the value of x when $y = 0$.

$$3x - 4y + 12 = 0 \quad \text{Original equation}$$

$$3x - 4(0) + 12 = 0 \quad \text{Substitute 0 for } y.$$

$$3x = -12 \quad \text{Subtract 12 from each side.}$$

$$x = -4 \quad \text{Divide each side by 3.}$$

The x -intercept is -4 . The graph crosses the x -axis at $(-4, 0)$.

Likewise, the y -intercept is the value of y when $x = 0$.

$$3x - 4y + 12 = 0 \quad \text{Original equation}$$

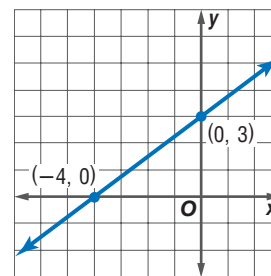
$$3(0) - 4y + 12 = 0 \quad \text{Substitute 0 for } x.$$

$$-4y = -12 \quad \text{Subtract 12 from each side.}$$

$$y = 3 \quad \text{Divide each side by } -4.$$

The y -intercept is 3 . The graph crosses the y -axis at $(0, 3)$.

Use these ordered pairs to graph the equation.



CHECK Your Progress

4. Find the x -intercept and the y -intercept of the graph of $2x + 5y - 10 = 0$. Then graph the equation.

CHECK Your Understanding

Example 1 (p. 66)

State whether each equation or function is linear. Write *yes* or *no*. If no, explain your reasoning.

1. $x^2 + y^2 = 4$

2. $h(x) = 1.1 - 2x$

Example 2 (p. 67)

ECONOMICS For Exercises 3 and 4, use the following information.

On January 1, 1999, the euro became legal tender in 11 participating countries in Europe. Based on the exchange rate on one particular day, the linear function $d(x) = 0.8881x$ could be used to convert x euros to U.S. dollars.

3. On that day, what was the value in U.S. dollars of 200 euros?

4. On that day, what was the value in euros of 500 U.S. dollars?

Example 3 (p. 67)

Write each equation in standard form. Identify A , B , and C .

5. $y = 3x - 5$

6. $4x = 10y + 6$

7. $y = \frac{2}{3}x + 1$

Example 4 (p. 68)

Find the x -intercept and the y -intercept of the graph of each equation. Then graph the equation.

8. $y = -3x - 5$

9. $x - y - 2 = 0$

Exercises

HOMEWORK HELP	
For Exercises	See Examples
10–17	1
18–21	2
22–27	3
28–33	4

State whether each equation or function is linear. Write *yes* or *no*. If no, explain your reasoning.

10. $x + y = 5$

11. $f(x) = 6x - 19$

12. $f(x) = 7x^5 + x - 1$

13. $h(x) = 2x^3 - 4x^2 + 5$

14. $g(x) = 10 + \frac{2}{x^2}$

15. $\frac{1}{x} + 3y = -5$

16. $x + \sqrt{y} = 4$

17. $y = \sqrt{2x - 5}$

PHYSICS For Exercises 18 and 19, use the following information.

When a sound travels through water, the distance y in meters that the sound travels in x seconds is given by the equation $y = 1440x$.

18. How far does a sound travel underwater in 5 seconds?

19. In air, the equation is $y = 343x$. Does sound travel faster in air or water? Explain.

ATMOSPHERE For Exercises 20 and 21, use the following information.

Suppose the temperature T in $^{\circ}\text{F}$ above the Earth's surface is given by $T(h) = -3.6h + 68$, where h is the height (in thousands of feet).

20. Find the temperature at a height of 10,000 feet.

21. Find the height if the temperature is -58°F .

Write each equation in standard form. Identify A , B , and C .

22. $y = -3x + 4$

23. $y = 12x$

24. $x = 4y - 5$

25. $x = 7y + 2$

26. $5y = 10x - 25$

27. $4x = 8y - 12$

Find the x -intercept and the y -intercept of the graph of each equation. Then graph the equation.

28. $5x + 3y = 15$

29. $2x - 6y = 12$

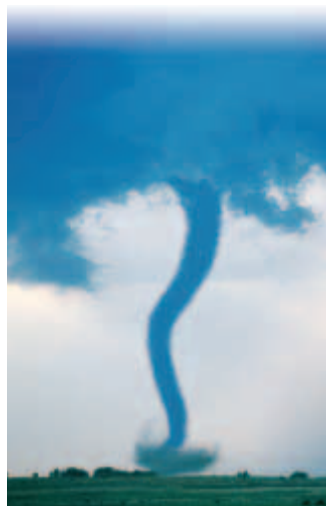
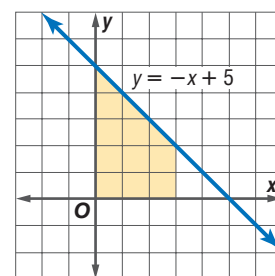
30. $3x - 4y - 10 = 0$

31. $2x + 5y - 10 = 0$

32. $y = x$

33. $y = 4x - 2$

34. **GEOMETRY** Find the area of the shaded region in the graph. (Hint: The area of a trapezoid is given by $A = \frac{1}{2}h(b_1 + b_2)$.)



Real-World Link

The troposphere is the lowest layer of the atmosphere. All weather events take place in the troposphere.

Write each equation in standard form. Identify A , B , and C .

35. $\frac{1}{2}x + \frac{1}{2}y = 6$

36. $\frac{1}{3}x - \frac{1}{3}y = -2$

37. $0.5x = 3$

38. $0.25y = 10$

39. $\frac{5}{6}x + \frac{1}{15}y = \frac{3}{10}$

40. $0.25x = 0.1 + 0.2y$

Find the x -intercept and the y -intercept of the graph of each equation. Then graph the equation.

41. $y = -2$

42. $y = 4$

43. $x = 8$

44. $3x + 2y = 6$

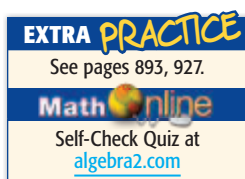
45. $x = 1$

46. $f(x) = 4x - 1$

47. $g(x) = 0.5x - 3$

48. $4x + 8y = 12$

49. **ATMOSPHERE** Graph the linear function in Exercises 20 and 21.



COMMISSION For Exercises 50–52, use the following information.

Latonya earns a commission of \$1.75 for each magazine subscription she sells and \$1.50 for each newspaper subscription she sells. Her goal is to earn a total of \$525 in commissions in the next two weeks.

50. Write an equation that is a model for the different numbers of magazine and newspaper subscriptions that can be sold to meet the goal.
51. Graph the equation. Does this equation represent a function? Explain.
52. If Latonya sells 100 magazine subscriptions and 200 newspaper subscriptions, will she meet her goal? Explain.

H.O.T. Problems

53. **OPEN ENDED** Write an equation of a line with an x -intercept of 2.

54. **REASONING** Explain why $f(x) = \frac{x+2}{2}$ is a linear function.

CHALLENGE For Exercises 55 and 56, use $x + y = 0$, $x + y = 5$, and $x + y = -5$.

55. Graph the equations. Then compare and contrast the graphs.
56. Write a linear equation whose graph is between the graphs of $x + y = 0$ and $x + y = 5$.
57. **REASONING** Explain why the graph of $x + 3y = 0$ has only one intercept.
58. *Writing in Math* Use the information about study time on page 66 to explain how linear equations relate to time spent studying. Explain why only the part of the graph in the first quadrant is shown and an interpretation of the graph's intercepts in terms of the situation.

STANDARDIZED TEST PRACTICE

59. **ACT/SAT** Which function is linear?

- A $f(x) = x^2$
B $g(x) = 2.7$
C $f(x) = \sqrt{9 - x^2}$
D $g(x) = \sqrt{x - 1}$

60. **REVIEW** What is the complete solution to the equation?

$$|9 - 3x| = 18$$

- F $x = -9; x = 3$ H $x = -3; x = 9$
G $x = -9; x = -3$ J $x = 3; x = 9$

Spiral Review

State the domain and range of each relation. Then graph the relation and determine whether it is a function. (Lesson 2-1)

61. $\{(-1, 5), (1, 3), (2, -4), (4, 3)\}$

62. $\{(0, 2), (1, 3), (2, -1), (1, 0)\}$

Solve each inequality. (Lesson 1-6)

63. $-2 < 3x + 1 < 7$

64. $|x + 4| > 2$

65. **TAX** Including a 6% sales tax, a paperback book costs \$8.43. What is the price before tax? (Lesson 1-3)

GET READY for the Next Lesson

PREREQUISITE SKILL Find the reciprocal of each number.

66. -4

67. $\frac{1}{2}$

68. $3\frac{3}{4}$

69. -1.25